NONLINEAR OPTICS (PHYC/ECE 568)

Spring 2022 - Instructor: M. Sheik-Bahae

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Homework #3, Due: Monday March 7

Problem 1. *Infrared Up-Conversion*

Estimate the efficiency of the upconversion of 10 μm infrared radiation using sum frequency generation. The pump laser has λ=532 nm, with a power of 10 W. Use 2 cm long Proustite (use $d_{eff}=d_{22}$) crystal under perfect phase-matching and optimum focusing where $L=2Z_0$ (Z_0 is the Rayleigh range of the focused laser beam).

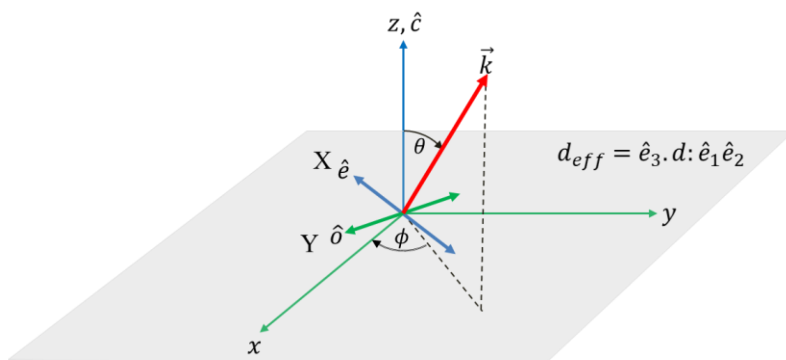
Problem 2.

(a) Show that d_{eff}, as defined by $P_3 = 4d_{eff} E_1 E_2$, is related to the *d* tensor via:

$$d_{eff} = \hat{e}_3 \bullet d{:}\hat{e}_1\hat{e}_2$$

where $\hat{e}_j$ *(j=1,2,3)* is the unit vector associated with E_1, E_2, and P_3.

(b) For a given geometry, d_{eff} is usually calculated in terms of d_{il}*'s* and the angles ϕ and θ as described in the figure below. Here *x, y and z* (or *1,2 and 3*) are the crystal axis and X, Y, and Z (laboratory frame) are optical propagation axis (e.g. $k_2 \| k_1 \| Z$). Note: Y is in xy-plane (thus normal to z- or optics axis) and X is on zZ plane.



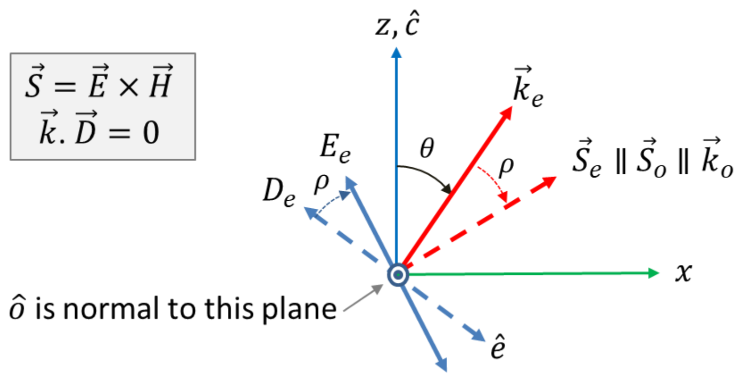
(i) Derive expressions for d_{eff} for a class 3m crystal (e.g. $LiNbO_3$) where $\hat{e}_1 = \hat{e}_2 = Y$ (ordinary), and $\hat{e}_3 = X$ (extra-ordinary). (This is known as type-I condition).

(ii) Repeat the above calculation for type-II condition where $\hat{e}_1 = Y$(ordinary), $\hat{e}_2 = X$ and $\hat{e}_3 = X$ (extra-ordinary).

(iii) Find a geometry (i.e. θ and ϕ) that accesses the largest d_{il} element in $LiNbO_3$. (see data provided here).

(iv) Find the phase matching angle for SHG generation at λ (fundamental)=1.15 μm for the part (i) and (ii). Discuss the phase matching situation for case (iii)

Problem 3. *Poynting Vector Walk-off:*
We know, from linear optics, that the e- and o-rays in a birefringent crystal walk-off from each other (i.e. double-refraction) resulting from the fact that k_e and k_o are not parallel. Known as Poynting vector walk-off, this is essentially the angle (ρ) between E and D vectors for the e-ray where D=ϵ: E.



In the harmonic generation applications, such as SHG, this imposes a serious restriction on the useful length of the nonlinear crystal.

a. Assuming a uniaxial crystal, calculate the walk-off angle ρ between e- and o-ray Poynting vectors. Show that

$$\tan\rho = -\frac{1}{n_e(\theta)}\frac{dn_e(\theta)}{d\theta}$$

b. Show that for type-I phase matching SHG (o+o→e)

$$\rho \approx \tan\rho = \frac{n_o^2(\omega)}{2}\left[\frac{1}{\tilde{n}_e^2(2\omega)} - \frac{1}{n_o^2(2\omega)}\right]\sin(2\theta_m)$$

$LiNbO_3$ Properties

$$\begin{bmatrix} 0 & 0 & 0 & 0 & d_{31} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{31} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix}$$ ***class 3m crystal***

Nonlinear Optical Coefficients of $LiNbO_3$ at 1.06 µ m

d_{22} / l d_{36}^{KDP} l	6.5
d_{31} / l d_{36}^{KDP} l	-12.3
d_{33} / l d_{36}^{KDP} l	-86

d_{36} (KDP). = 0.4 pm/V

Refractive Indices at 20°C

Wavelength, µm	n_o	n_e
0.43584	2.39276	2.29278
0.54608	2.31657	2.22816
0.63282	2.28647	2.20240
1.1523	2.2273	2.1515
3.3913	2.1451	2.0822